RISHAN SUNN

LECTURE PLAN

PAPER VII-STATISTICAL ECONOMICS

Regression line or Regression equation

Regression line or Regression equation is the equation of two or more variable. With the help of this Regression line or Regression equation we can estimate the unknown value from the known value or we can estimate the dependent variable from the independent variable.

If y is a dependent variable and x is a independent variable or we can say if y depend on x, we get a regression line or Regression equation of y on x, given by

$$y - \bar{y} = byx(x - \bar{x})$$

Where $\bar{y}$: is the mean value of y

$\bar{x}$ = is the mean value of x

byx = Regression coefficient of y on x

Similarly,

If x is a dependent variable and y is a independent variable or if x depend on y, we have a Regression line or Regression equation ~~x~~ of x on y, given by

$$x - \bar{x} = bxy(y - \bar{y})$$

Here ~~also~~ also

$\bar{x}$ is the mean value of x

$\bar{y}$ is the mean value of y

and bxy is the Regression coefficient of x on y

Q. Fit the Regression line from the following data

x: 3 5 10

y: 1 4 7

Sol^n

x	y	x^2	y^2	xy
3	1	9	1	3
5	4	25	16	20
10	7	100	49	70
$\Sigma x = 18$	$\Sigma y = 12$	$\Sigma x^2 = 134$	$\Sigma y^2 = 66$	$\Sigma xy = 93$

$n = 3$

$\bar{x} = \frac{\Sigma x}{n} = \frac{18}{3} = 6$ | $\bar{y} = \frac{\Sigma y}{n} = \frac{12}{3} = 4$

$\bar{x} = 6, \ \bar{x}^2 = 36$ | $\bar{y} = 4, \ \bar{y}^2 = 16$

Now, $b_{yx} = \dfrac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\frac{1}{n}\Sigma x^2 - \bar{x}^2} = \dfrac{\frac{1}{3}93 - 6\times 4}{\frac{1}{3}134 - 36}$

$= \dfrac{31 - 24}{44.67 - 36} = \dfrac{7}{8.67}$

$b_{yx} = 0.81$

$b_{xy} = \dfrac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\frac{1}{n}\Sigma y^2 - \bar{y}^2} = \dfrac{7}{\frac{1}{3}66 - 16} = \dfrac{7}{6}$

$b_{xy} = 1.17$

To fit the regression line of y on x, now

$y - \bar{y} = b_{yx}(x - \bar{x})$

$y - 4 = 0.81(x - 6)$

$y - 4 = 0.81x - 4.86$

$y = 0.81x - 4.86 + 4$

$y = 0.81x - 0.86$ which is the regression line of y on x

Q The age of the father and the age of the son are given below.

Age of father: 19 20 23 25 30
Age of Son 1 3 5 7 10

estimate the ~~linear~~ Regression equation of the age of the father on that of the age of the son.
What would be the expected age of the father if the age of son was 15 years..

Sol^n Let the age of the father be x
the age of the son be y

x	y	xy	y^2
19	1	19	1
20	3	60	9
23	5	115	25
25	7	175	49
30	10	300	100
$\Sigma x = 117$	$\Sigma y = 26$	$\Sigma xy = 669$	$\Sigma y^2 = 184$

$n = 5$, $\bar{x} = \frac{\Sigma x}{n} = \frac{117}{5} = 23.4$

$\bar{y} = \frac{\Sigma y}{n} = \frac{26}{5} = 5.2$

$\bar{y}^2 = 27.04$

The Regression coefficient of the age of father on that of the age of the son of the regression coefficient of x on y is given by

$$bxy = \frac{\frac{1}{n}\Sigma xy - \bar{x}\bar{y}}{\frac{1}{n}\Sigma y^2 - \bar{y}^2}$$

$$bxy = \frac{\frac{1}{5}669 - 23.4 \times 5.2}{\frac{1}{5}184 - 27.04}$$

$$bxy = \frac{12.12}{9.76} = 1.24$$

Now, the Regression equation of the age of the father on that the age of the son or the Regression equation of x on y is given by

$$x - \bar{x} = bxy(y - \bar{y})$$

$$x - 23.4 = 1.24(y - 5.2)$$

$$x - 23.4 = 1.24y - 6.45$$

$$x = 1.24y - 6.45 + 23.4$$

$x = 1.24y + 16.95$ which is the Regression equation of the age of the father on the age of the son.

Next,

Given the age of the son, that is

$$y = 15$$

$$x = ?$$

Substitute the value of y on the Regression equation, we have

$$x = 1.24y - 16.95$$

$$x = 1.24(15) - 16.95$$

$$x = 35.6$$

Hence the age of the father is $x = 35.6$

Similarly,

To fit the regression line of x on y, now

$$x - \bar{x} = b_{xy}(y - \bar{y})$$

$$x - 6 = 1.17(y - 4)$$

$$x - 6 = 1.17y - 4.68$$

$$x = 1.17y - 4.68 + 6$$

$x = 1.17y + 1.32$ which is the regression line of x on y.

Q1 From the data below fit the two regression lines

X:	2	4	6	8	10
Y:	5	7	9	8	11

Q2 Given

X	1	5	3	2	1	1	7	3
Y	6	1	0	0	1	2	1	5

(a) Fit the regression line of Y on X

(b) Fit the regression line of X on Y

(c) calculate the correlation coefficient between X and Y.

Q 3 The mark obtain by boys and girls are Given below -

marks of boys :	50	51	56	60	63
marks of girls :	45	52	60	65	70

estimate the Regression equation of the marks of girls on that of the marks of boys. What would be the expected mark of girls if the marks of boys is 76 ?

Q 4. The following data give the ages and blood pressure of 10 women -

age :	56	42	36	47	49	42	60	72	63	55
Blood pressure :	147	125	118	128	145	140	155	160	149	150

(i) Determine the correlation coefficient between the ages and blood pressure

(ii) Determine the regression equation of blood pressure on that of ages

(iii) Estimate the blood pressure of a woman whose age is 45 years.

Related concept of Probability

For a clear understanding of probability and its various rules, it is necessary to introduce some related concept and term of probability.

(1) Trial - Any experiment can be called trial. eg. Tossing of a coin is a trial, also throwing of a dice (dudo dice) is a trial.

(2) Events - The possible outcome in a trial are called events. for eg - In tossing a coin head and tail are the two possible outcome, Hence head and tail are called events.

(3) Favourable Events - In a trial, the event we want to occur or the events which is favourable to occur is called favourable events and the number of cases related to the favourable events determine the favourable cases.

(4) Exhaustive cases - In a trial, the total possible outcome determine the exhaustive cases. eg, in tossing of a coin, there are two outcome, Hence the exhaustive cases is two. And in throwing of dice, there are six possible outcome, Hence the exhaustive cases is six.

(5) Mutually exclusive event - In any trial which can occur simultaneously are called mutually exclusive events. eg tossing of a coine and throwing of dice.

(6) Equally likely cases - Two or more events are said tobe equally likely if the chance of their happening is equal, that is, there is no preference of any one over other. Thus in throwing of a dice, the coming out of 1, 2, 3, 4, 5 or 6 is equally likely. In tossing the coming out head or tail is equally likely.

) Independent and dependent events –
An event is said to be independent if it happenning is not affected by the happenning of other event.
eg throwing of a dice Repeatedly, comming out of five in a first throw in independent of comming up again in second throw.

However, if we are successively drawing a card from the pack of cards (playing cards) with out replacement the event will be dependent.

eg. The chance of getting a king on is $4/52$. If this card is not replace before the second draw the ~~chang~~ chance of getting the king is $3/51$

Probability

It is define in term of event. Let A be the event. The probability of A denoted by P(A) and it is define as

$$P(A) = \frac{\text{favourable cases for } A}{\text{Exhaustive cases of } A}$$

~~eg a coin is tosse~~

(1) eg → A coin is tosse, find the probability of getting a head.

Solⁿ Total outcome = 2 (Head and tail)

The event we want to occure is head, So the favourable case = 1

let A be the event of getting a head

$\therefore P(A) = \frac{1}{2} = 0.50$

* It mean ~~to~~ tele that because there is only two outcome the chance of getting a head or tail is 50/50

Theorem of probability

Let A and B be the two events, the probability of getting A or B or both $(A \cup B)$ is given by

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Where $A \cup B$ = A union B

$A \cap B$ = A intersection B

If A and B are mutually exclusive events or independent events, then

$P(A \cap B) = 0$, hence we have

$$P(A \cup B) = P(A) + P(B)$$

This is know as the addition law of probability and it is state that the probability of the sum of two mutually exclusive events is equal to the sum of their respective probability.

Theorem of a compliment of probability

If A is the event of getting A, then A^c is the event of not getting A, we can say that the event A is compliment to A^c. If A^c contain all the element which are not belong to A

$$P(A^c) = 1 - P(A)$$

$$P(A) = 1 - P(A^c)$$

$$P(A) + P(A^c) = 1$$

Conditional Probability

Let A and B be the two events, the probability of getting A when B has already happened is know as the conditional probability of A Given B. It is denoted by $P(A/B)$ and it defind as

$$P(A/B) = \frac{P(AB)}{P(B)}, \quad P(B) \neq 0$$

Similarly, the probability of getting B when A has already happen is know as the conditional probability of B Given A. It is denoted as $P(B/A)$ and defined as $P(B/A) = \frac{P(AB)}{P(A)}, \quad P(A) \neq 0$

If A and B are independent or mutually exclusive events

$$P(A/B) = P(A)$$

$$P(B/A) = P(B)$$

By defination of conditional probability

$$P(A/B) = \frac{P(AB)}{P(B)} \quad \text{—— (1)}$$

If A and B are independent events

$$P(A/B) = P(A)$$

From (1) $P(A) = \frac{P(AB)}{PB}$

$$P(AB) = P(A).P(B)$$

This is know as the multiplicative law of probability

Q If $P(A) = .4$, $P(B) = .8$
and $P(AB) = .2$
find $P(A+B)$.

Sol^N $P(A+B)$ or $P(A \cup B) = P(A) + P(B) - P(AB)$

$= .4 + .8 - .2$

$= 1.2 - .2$

$P(A+B) = 1$

Q Given $P(A+B) = \frac{5}{6}$, $P(AB) = \frac{1}{3}$ and $P(A^c) = 1/2$
find $P(A)$ and $P(B)$ and show that A and B are independent events.

Sol^N $P(A+B) = P(A) + P(B) - P(AB)$ —— ①

$P(A^c) = 1 - P(A)$

$\frac{1}{2} = 1 - P(A)$

$P(A) = 1 - \frac{1}{2} = \frac{1}{2}$

From ①

$\frac{5}{6} = \frac{1}{2} + P(B) - \frac{1}{3}$

$\frac{5}{6} - \frac{1}{2} + \frac{1}{3} = P(B)$

$\frac{5-3+2}{6} = P(B)$

$P(B) = \frac{2}{3}$

To show that A and B are independent event, we have to prove that $P(AB) = P(A) \cdot P(B)$

LHS $\Rightarrow P(AB) = \frac{1}{3}$

RHS $\Rightarrow P(A) \cdot P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$

Hence we proved that

Poisson Distribution

The probability mass function of poisson distribution is given by

$$P(x) = \frac{e^{-\lambda} \lambda^{x}}{\lfloor x}$$

Where λ = Mean = np

e = 2.71828 base on the natural system of logarithm

x = probability of r success

eg → The probability of 0, 1, 2, 3 and 4 would be

Success (x)	Probability $P(x)$
0	$e^{-\lambda}$
1	$\frac{e^{-\lambda} \lambda^{1}}{\lfloor 1}$
2	$\frac{e^{-\lambda} \lambda^{2}}{\lfloor 2}$
3	$\frac{e^{-\lambda} \lambda^{3}}{\lfloor 3}$
4	$\frac{e^{-\lambda} \lambda^{4}}{\lfloor 4}$

Fitting of a poisson distribution

Fit a poisson distribution to the following data and calculate the theoritical or expected frequency

x:	0	1	2	3	4
f:	123	59	14	3	1

Soln

Calculation of mean (λ)

x	f	fx
0	123	0
1	59	59
2	14	28
3	3	9
4	1	4
	$N=200$	$\Sigma fx = 100$

$$\bar{x} = \lambda = \frac{\Sigma fx}{N}$$

$$= \frac{100}{200}$$

$$\lambda = 0.5$$

Fitting of a poisson distribution

x	$P(x)$	$NP(x)$
0	$P(0) = e^{-\lambda} = e^{-0.5} = 0.6065$	$200 \times 0.6065 = 121$
1	$P(1) = \frac{e^{-0.5}\, 0.5^1}{\lfloor 1} = 0.3033$	$200 \times 0.3033 = 61$
2	$P(2) = \frac{e^{-0.5}(0.5)^2}{\lfloor 2} = 0.0758$	$200 \times 0.0758 = 15$
3	$P(3) = \frac{e^{-0.5}(0.5)^3}{\lfloor 3} = 0.126$	$200 \times 0.126 = 3$
4	$P(4) = \frac{e^{-0.5}(0.5)^4}{\lfloor 4} = 0.0016$	$200 \times 0.0016 = 0$ (APX)

$\Sigma NP(x) = 200$ (APX)

The last column is fitting of poisson distn or calculation of theoritical or expected frequency.

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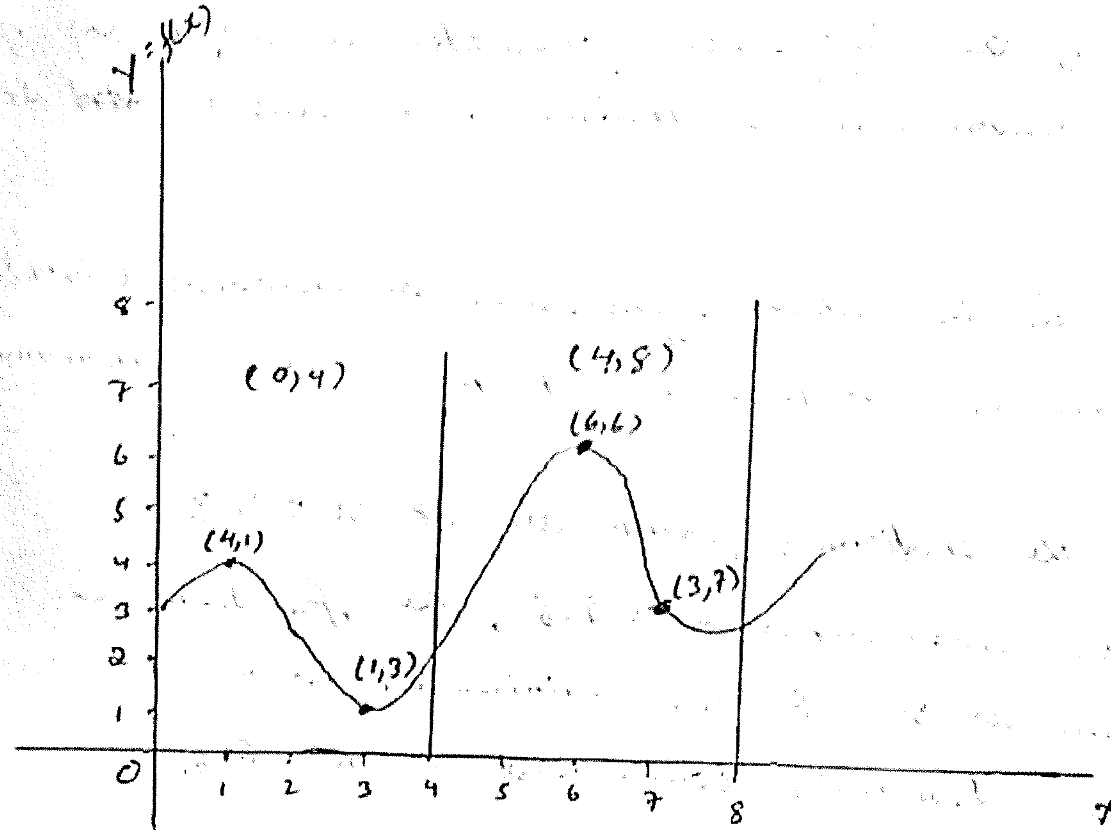
LECTURE PLAN

PAPER IV-MATHEMATICAL ECONOMICS

Maxima and Minima

Function is nothing but dependent valiable, i,e, value of function is nothing but value of dependent valiable.

To detelmine the maxima and minima of a functio we filst consider a class intelval.



From the above figure, by considelling the intelval (0,4), the greatest value of the function, i,e, y = 4, this is Know as the maximum value of the function.

Within the Same class intelval, the least value of the function, i,e, y = 1, this is Know as the minimum value of the function.

is $y = 6$, this is know.

Also within this ~~as~~ interval the minimum value of the function

is $y = 3$.

From the above analysis, we have ~~two~~ two maximum value

as well as ~~two~~ two minimum.

The value of the independent variables at which the function is either maximum or minimum is called ~~sta~~ stationary points.

As show in the above figure, for the interval $(0, 4)$ the function is maximum at $x = 1$ and minimum at $x = 3$.

Therefore the stationary point are at $x = 1, 3$

For the class interval $(4, 8)$, the function is maximum at $x = 6$ and minimum at $x = 7$.

To calculate the maxima and minima of a function, we have to follow the following step.

1) Find $\frac{dy}{dx}$

2) Equate $\frac{dy}{dx} = 0$, this is know as the necessaly condition of the first older condition for maxima or minima. From this condition we get the value(s) of independent valiable(s), i.e, the stationaly point.

3) Find $\frac{d^2y}{dx^2}$

If $\frac{d^2y}{dx^2} > 0$, the function is minimum at the stationaly point

If $\frac{d^2y}{dx^2} < 0$, the function is maximum at that stationaly point.

If $\frac{d^2y}{dx^2} = 0$, we can not determine the maxim and minimum value of the function. We are at the ~~inflex~~ inflection point.

etel mine the stationaly point from the following functi

$y = x^2 - 6x + 5$

1^n Given $y = x^2 - 6x + 5$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 6x + 5)$$

$$= \frac{d}{dx}x^2 - \frac{d}{dx}6x + \frac{d}{dx}5$$

$$= 2x - 6\frac{d}{dx}x + 0$$

$$= 2x - 6(1)$$

$$\frac{dy}{dx} = 2x - 6$$

According to the neccesaly condition fol maxima & minima

$$\frac{dy}{dx} = 0$$

i,e $2x - 6 = 0$

$$2x = 6$$

$$x = 6/2$$

$$x = 3$$

Hence, the stationaly point is at $x = 3$

(2) $y = \frac{1}{3}x^3 - 3x^2 + 8x + 10$

Given $y = \frac{1}{3}x^3 - 3x^2 + 8x + 10$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{3}x^3 - 3x^2 + 8x + 10\right)$$

$$= \frac{d}{dx}\frac{1}{3}x^3 - \frac{d}{dx}3x^2 + \frac{d}{dx}8x + \frac{d}{dx}10$$

$$= \frac{1}{3}\frac{d}{dx}x^3 - 3\frac{d}{dx}x^2 + 8\frac{d}{dx}x + 0$$

$$= \frac{1}{\cancel{3}}(\cancel{3})x^{3-1} - 3(2)x + 8(1)$$

$$\frac{dy}{dx} = x^2 - 6x + 8$$

According to the neccesary condition for maxima and minima

$$\frac{dy}{dx} = 0$$

i,e $x^2 - 6x + 8 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 32}}{2}$$

$$x = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2}$$

$$x = \frac{6+2}{2}, \frac{6-2}{2}$$

$$x = 4, 2$$

Hence the stationaly point are at $x = 4, 2$

Determine the maxima / minima of the following functions

① $y = x^2 - 12x + 10$

Solⁿ Given $y = x^2 - 12x + 10$

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 - 12x + 10)$$

$$= \frac{d}{dx}x^2 - \frac{d}{dx}12x + \frac{d}{dx}10$$

$$= 2x - 12\frac{d}{dx}x + 0$$

$$\frac{dy}{dx} = 2x - 12$$

according to the necessary condition for maxima and minima

$$\frac{dy}{dx} = 0, \text{ i,e, } 2x - 12 = 0$$

$$2x = 12$$

$$x = \frac{12}{2}$$

$$x = 6$$

∴ stationary point is at $x = 6$

according to the sufficient condition for maxima and mini

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(2x - 12)$$

$$= \frac{d}{dx}2x - \frac{d}{dx}12$$

$$= 2\frac{d}{dx}x - 0$$

$$\frac{dy}{dx} = 2 > 0$$

Hence the function is minimum at $x = 6$

$y = x^3 - 2x^2 + 1$

$y = x^3 - 2x^2 + 1$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 - 2x^2 + 1)$$

$$= \frac{d}{dx}x^3 - \frac{d}{dx}2x^2 + \frac{d}{dx}1$$

$$= 3x^2 - 2\frac{d}{dx}x^2 + 0$$

$$= 3x^2 - 2(2)x$$

$$\frac{dy}{dx} = 3x^2 - 4x$$

according to the necessary condition for maxima and minima

$$\frac{dy}{dx} = 0, \quad i.e, \quad 3x^2 - 4x = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\times3\times0}}{2\times3}$$

$$x = \frac{4 \pm \sqrt{16 - 0}}{6}$$

$$x = \frac{4 \pm \sqrt{16}}{6} = \frac{4 \pm 4}{6}$$

$$x = \frac{4+4}{6}, \frac{4-4}{6}$$

$$x = \frac{4}{3}, 0$$

∴ the stationaly point are at $x = 4/3, 0$

According to the sufficient condition for maxima and minima

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$= \frac{d}{dx}(3x^2 - 4x)$$

$$= \frac{d}{dx}3x^2 - \frac{d}{dx}4x$$

$$= 3\frac{d}{dx}x^2 - 4\frac{d}{dx}x$$

$$= 3(2)x - 4(1)$$

$$\frac{d^2y}{dx^2} = 6x - 4$$

Here we have two stationary point at $x = 4/3, 0$

First At $x = 4/3$

$$\frac{d^2y}{dx^2} = 6x - 4$$

$$= \cancel{6}^{2}(4/\cancel{3}) - 4$$

$$= 8 - 4$$

$$\frac{d^2y}{dx^2} = 4 > 0$$

$\therefore$ the function is minimum at $x = 4/3$

Next At $x = 0$

$$\frac{d^2y}{dx^2} = 6x - 4$$

$$= 6(0) - 4$$

$$= 0 - 4$$

$$\frac{d^2y}{dx^2} = -4 < 0$$

Hence the function is maximum at $x = 0$

Application of Derivative in Economics

As in mathematics in economics also we have many functions such as Revenue function, cost function, profit function etc.

In almost all economics function, the quantity (q) is the independent variable.

$R = f(q) \rightarrow$ Revenue function

$C = f(q) \Rightarrow$ Cost function

$\pi = f(q) \rightarrow$ Profit function

$AC = f(q) \rightarrow$ Average cost function

$MC = f(q) \rightarrow$ Marginal cost function

$AR = f(q) \rightarrow$ Average Revenue function

$MR = f(q) \rightarrow$ Marginal Revenue function

and so on

The demand function or the price function can be expressed in either of the following forms

$P = f(q) \rightarrow$ Price function

or $q = f(P) \rightarrow$ Demand function

But for uniformity, the demand function is given by $P = f(q)$

Formula

(i) $C = AC \times Q$

(ii) $R = AR \times Q$ or $P \times Q$

(iii) $AC = \frac{C}{Q}$

(iv) $MC = \frac{dC}{dQ}$ (where d is denoted as chang

(v) $AR = \frac{R}{Q}$

(vi) $MR = \frac{dR}{dQ}$

(vii) $\pi = R - C$

Q.1 The demand function is given by

$$P = 100 - q$$

Write down the Revenue function

Soln Given $P = 100 - q$

$$R = P \times q$$

$$R = (100 - q)q$$

$R = 100q - q^2$ which is a Revenue function

Q.2 The demand function is given by

$$P = 2q^2 - 3q$$

Write down the MR and AR function

Soln Given $P = 2q^2 - 3q$

$$MR = \frac{dR}{dq}$$

$$R = P \times q$$

$$R = (2q^2 - 3q)q$$

$$R = 2q^3 - 3q^2$$

$$MR = \frac{dR}{dq} = \frac{d}{dq}(2q^3 - 3q^2)$$

$$= \frac{d}{dq}2q^3 - \frac{d}{dq}3q^2$$

$$= 2\frac{d}{dq}q^3 - 3\frac{d}{dq}q^2$$

$$= 2(3)q^{3-1} - 3(2)q^{2-1}$$

$MR = 6q^2 - 6q$ which is a marginal Revenue function

$$AR = \frac{R}{q} = \frac{2q^3 - 3q^2}{q}$$

$AR = 2q^2 - 3q$ Which is a Average Reven function.

Q3 Given the cost function write down the AC and MC function

$$C = q^2 - 3q + 4$$

Given $C = q^2 - 3q - 4$

$$MC = \frac{dC}{dq} = \frac{d}{dq}(q^2 - 3q + 4)$$

$$= \frac{d}{dq}q^2 - \frac{d}{dq}3q + \frac{d}{dq}4$$

$$= 2q - 3\frac{d}{dq}q + 0$$

$$= 2q - 3(1)$$

$MC = 2q - 3$ Which is a MC function

$$AC = \frac{C}{q} = \frac{q^2 - 3q + 4}{q}$$

$AC = q - 3 + \frac{4}{q}$ Which is a AC function

(4) Given the Revenue and cost function

$$R = q^2 + 3q - 9$$
$$C = 3q^3 + q^2 - 3q$$

Write down the profit function

Sol^n Given $R = q^2 + 3q - 9$
$C = 3q^3 + q^2 - 3q$

$$\pi = R - C$$
$$= (q^2 + 3q - 9) - (3q^3 + q^2 - 3q)$$
$$= q^2 + 3q - 9 - 3q^3 - q^2 + 3q$$

$\pi = -3q^3 + 6q - 9$ which is the profit function

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5) Given the demand function and the cost function

$$P = 3q^2 - 2q$$
$$C = q^3 - 3q^2 + 4$$

Write down the profit function

Sol^n Given $P = 3q^2 - 2q$
$C = q^3 - 3q^2 + 4$

$$\pi = R - C$$
$$R = P \times q = (3q^2 - 2q)q$$
$$R = 3q^3 - 2q^2$$
$$\pi = (3q^3 - 2q^2) - (q^3 - 3q^2 + 4)$$
$$= 3q^3 - 2q^2 - q^3 + 3q^2 - 4$$

$\pi = 2q^3 + q^2 - 4$ which is a profit function

Partial Derivative

Let $u = f(x, y)$ be a function. The derivative of u with respect to x ~~can~~ canbe determined by treating y as a constant. In this case we get the partial derivative of u with respect to x and it is denoted by $\frac{\partial u}{\partial x}$.

Similarly, the derivative of u with respect to y canbe determined by treating x as a constant. In this case we get the partial derivative of u with respect to y and is denoted by $\frac{\partial u}{\partial y}$.

eg $u = x^2 y^3 + 3xy^2$

sol^n $\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2 y^3 + 3xy^2)$

$= \frac{\partial}{\partial x} x^2 y^3 + \frac{\partial}{\partial x} 3xy^2$ (Here y is a constant)

$= y^3 \frac{\partial}{\partial x} x^2 + 3y^2 \frac{\partial}{\partial x} x$

$= y^3 \, 2x^{2-1} + 3y^2 (1)$

$\frac{\partial u}{\partial x} = 2y^3 x + 3y^2$

$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^2 y^3 + 3xy^2)$

$= \frac{\partial}{\partial y} x^2 y^3 + \frac{\partial}{\partial y} 3xy^2$ (Here x is a constant)

$= x^2 \frac{\partial}{\partial y} y^3 + 3x \frac{\partial}{\partial y} y^2$

$= x^2 \, 3y^{3-1} + 3x(2)y^{2-1}$

$\frac{\partial u}{\partial y} = 3x^2 y^2 + 6xy$

Let $u = f(x, y)$ be a function. There are two first order partial derivative, i.e.,

$$\frac{\partial u}{\partial x} \quad \text{and} \quad \frac{\partial u}{\partial y}$$

There are four second order partial derivative

(1) $$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

(2) $$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

(3) $$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$$

(4) $$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)$$

The last two (3 and 4) are called cross second order partial derivative and they are always equal, i.e., $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

~~de~~ partial derivative.

Sol^n Given $u = x^2y^4 - 3xy^5$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}(x^2y^4 - 3xy^5)$$

$$= \frac{\partial}{\partial x} x^2y^4 - \frac{\partial}{\partial x} 3xy^5$$

$$= y^4 \frac{\partial}{\partial x} x^2 - 3y^5 \frac{\partial}{\partial x} x$$

$$= y^4(2)x - 3y^5(1)$$

$$\frac{\partial u}{\partial x} = 2y^4x - 3y^5$$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}(x^2y^4 - 3xy^5)$$

$$= \frac{\partial}{\partial y} x^2y^4 - \frac{\partial}{\partial y} 3xy^5$$

$$= x^2 \frac{\partial}{\partial y} y^4 - 3x \frac{\partial}{\partial y} y^5$$

$$= x^2(4)y^{4-1} - 3x(5)y^{5-1}$$

$$\frac{\partial u}{\partial y} = 4x^2y^3 - 15xy^4$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}(2y^4x - 3y^5)$$

$$= \frac{\partial}{\partial x} 2y^4x - \frac{\partial}{\partial x} 3y^5$$

$$= 2y^4 \frac{\partial}{\partial x} x - 0$$

$$= 2y^4(1)$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)$$

$$= \frac{\partial}{\partial y}(4x^2y^3 - 15xy^4)$$

$$= \frac{\partial}{\partial y}4x^2y^3 - \frac{\partial}{\partial y}15xy^4$$

$$= 4x^2\frac{\partial}{\partial y}y^3 - 15x\frac{\partial}{\partial y}y^4$$

$$= 4x^2(3)y^{3-1} - 15x(4)y^{4-1}$$

$$\frac{\partial^2 u}{\partial y^2} = 12x^2y^2 - 60xy^3$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$$

$$= \frac{\partial}{\partial x}(4x^2y^3 - 15xy^4)$$

$$= \frac{\partial}{\partial x}4x^2y^3 - \frac{\partial}{\partial x}15xy^4$$

$$= 4y^3\frac{\partial}{\partial x}x^2 - 15y^4\frac{\partial}{\partial x}x$$

$$= 4y^3(2)x^{2-1} - 15y^4(1)$$

$$\frac{\partial^2 u}{\partial x \partial y} = 8y^3x - 15y^4$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = \frac{\partial}{\partial y}(2y^4x - 3y^5)$$

$$= \frac{\partial}{\partial y}2y^4x - \frac{\partial}{\partial y}3y^5$$

$$= 2x\frac{\partial}{\partial y}y^4 - 3\frac{\partial}{\partial y}y^5$$

$$= 2x(4)y^{4-1} - 3(5)y^{5-1}$$

$$\frac{\partial^2 u}{\partial y \partial x} = 8y^3x - 15y^4$$

Hence $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} = 8y^3x - 15y^4$